EXERCISE – II HINTS & SOLUTIONS

Sol.1 B,D

$a_1, a_2, ..., a_n, ...$

$a_2 = \frac{a_1 + a_3}{2} \Rightarrow a_1 + a_3 - 2a_2 = 0$

$a_1 - 2a_2 + a_3 = 0$

$-2(a_2 - 2a_3 + a_4) = 0$

$a_3 - 2a_4 + a_5 = 0$

add $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

Sol.2 A,C

$p + 1 + \frac{1}{p} + \frac{1}{p^2} + \frac{1}{p^3} + ... \infty = \frac{9}{2}$

$\Rightarrow \frac{p}{1 - \frac{1}{p}} = \frac{9}{2} \Rightarrow \frac{p^2}{p-1} = \frac{9}{2} \left| \begin{array}{l} \because r = \frac{1}{p} \\ -1 < r < 1 \end{array} \right.$

$\Rightarrow 2p^2 - 9p + 9 = 0$

$\Rightarrow (p - 3)(2p - 3) = 0$

$p = 3, p = \frac{3}{2}$

Sol.3 A,B

$2b = a + c$

$4b^2 = a^2 + c^2 + 2ac$

$\Rightarrow a^2 + c^2 = 4b^2 - 2ac$

& $b^2 = \frac{2a^2c^2}{a^2 + c^2}$

$\Rightarrow b^2 (4b^2 - 2ac) = 2a^2c^2$

$\Rightarrow b^2 (2b^2 - ac) = a^2c^2$

$\Rightarrow 2b^4 - b^2 (ac) - (ac)^2 = 0$

$\Rightarrow (b^2 - ac)(2b^2 + ac) = 0$

$\Rightarrow b^2 = ac$

$\Rightarrow$ a, b, c in G.P. & a, b, c in A.P.

$\Rightarrow a = b = c$

or $2b^2 + ac = 0 \Rightarrow b^2 = -\frac{ac}{2} \Rightarrow$ a, b, $\frac{-c}{2}$ in G.P.

Sol.4 A,B,C

a, b > 0 , a > b

A = 2G

$\frac{a+b}{2} = 2\sqrt{ab} \Rightarrow \frac{a+b}{\sqrt{ab}} = 4$

$\Rightarrow (a - b)^2 = 12ab \Rightarrow \frac{a-b}{\sqrt{ab}} = 2\sqrt{3}$

$\Rightarrow \frac{a+b}{a-b} = \frac{2}{\sqrt{3}}$ C & D apply

$\Rightarrow \frac{a+b+a-b}{a+b-a+b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

$\Rightarrow \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

Rationalize

$\Rightarrow \frac{a}{b} = \frac{(2+\sqrt{3})^2}{1} = \frac{7+4\sqrt{3}}{1} = \frac{1}{7-4\sqrt{3}}$

Sol.5 A,B,C,D

If $\sum_{r=1}^{n} r(r+1)(2r+3) = an^4 + bn^3 + cn^2 + dn + e$

$= \sum (2r^3 + 5r^2 + 3r)$

$= \frac{2n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}$

$= \frac{n(n+1)}{2} [n(n+1) + \frac{5}{3}(2n+1) + 3]$

$= \frac{n(n+1)}{6}\ [3n^2 + 3n + 10n + 5 + 9]$

$= \frac{n(n+1)}{6}[3n^2 + 13n + 14] = \frac{1}{6}\ [3n^4 + 16n^3 + 27n^2 + 14n]$

$= \frac{1}{2}\ n^4 + \frac{8}{3}n^3 + \frac{9}{2}n^2 + \frac{7}{3}\ n + 0$

a b c d e

(A) a + c = b + d, (B) e = 0

(D) $\frac{c}{a} = \frac{9}{2} \times \frac{2}{1} = 9 \in I$

(C) $a, b - \frac{2}{3}, c - 1 \Rightarrow \frac{1}{2}, 2, \frac{7}{2}$ in A.P.

Sol.6 A,B,C,D

$b_1, b_2, b_3 > 0$ in a G.P.

$b_2^2 = b_1 b_3$

$\Rightarrow \frac{b_2}{b_1} = \frac{b_3}{b_2} = r > 0$

$b_3 > 4b_2 - 3b_1 \quad \because \frac{b_1}{b_3} = \frac{1}{r^2}$

$\Rightarrow 1 > 4\ \frac{b_2}{b_3} - \frac{3b_1}{b_3} \Rightarrow 1 > \frac{4}{r} - \frac{3}{r^2} \quad \because r > 0$

$\Rightarrow r^2 - 4r + 3 > 0$

$\Rightarrow (r - 3)(r - 1) > 0$

$\Rightarrow r > 3$ or $0 < r < 1$

$\Rightarrow 3.5, 5.2 > 3$

Sol.7 A,C

$$\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx} + \sqrt{a+(r-1)x}}$$

$$= \sum_{1}^{n} \frac{\sqrt{a+rx} - \sqrt{a+(r-1)x}}{x}$$

$$= \frac{1}{x} \sum_{1}^{n} (\sqrt{a+rx} - \sqrt{a+(r-1)x})$$

$= \frac{1}{x}\ [\sqrt{a+nx}\ - \sqrt{a}\ ]$ Rationalize

$= \frac{1}{x}\ \frac{a+nx-a}{\sqrt{a+nx} + \sqrt{a}} = \frac{n}{\sqrt{a+nx} + \sqrt{a}}$

Sol.8 A,B,C

A.P., $2x = a + b$ & $x = y + 2$, $a = 5z$

G.P., $y^2 = ab$

H.P., $z = \frac{2ab}{a+b}$

$\Rightarrow z = \frac{2y^2}{2x} \Rightarrow y^2 = zx$

A.M. > G.M. > H.M.

$x > y > z$

$\frac{a}{5} = \frac{2ab}{a+b}$

$a^2 + ab = 10ab$

$\Rightarrow a(a - 9b) = 0$

$\Rightarrow a \neq 0$ or $a = 9b$

$y = x - 2$

$\therefore y = \frac{2x-4}{2} \Rightarrow y^2 = \frac{(a+b-4)^2}{4}$

$\Rightarrow a^2 + b^2 - 2ab - 8a - 8b + 16 = 0$

$\Rightarrow 4b^2 - 5b + 1 = 0$

$\Rightarrow (b - 1)(4b - 1) = 0$

$\Rightarrow b = 1, a = 9$

or $b = 1/4, a = 9/4$